Verification of Timed Asynchronous Programs

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Dec 13, FSTTCS 18, Ahmedabad

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Introduction

Asynchronous Programs

Widely used in building efficient and responsive software

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Jobs broken up into tasks that are assigned to parallel threads

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Jobs broken up into tasks that are assigned to parallel threads

Asynchronous tasks stored in a buffer, can execute later

Asynchronous execution can lead to extremely intricate and unpredictable behaviours programs.

Related Work

Most of the existing work on asynchronous programs considers the untimed version

[Sen, Viswanathan ’06] introduces multiset pushdown systems for recursive asynchronous programs

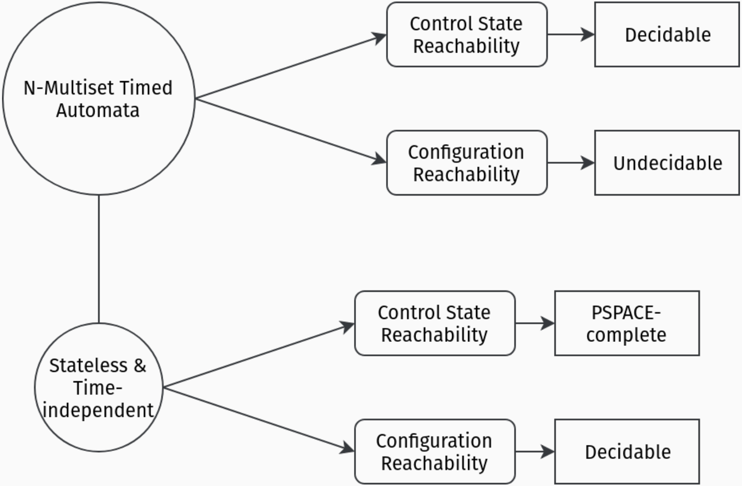
[Fang et al ’16] introduce timed task automata which are extensions of task automata[1]which have states associated with tasks and on triggering, it is added to a queue

In [Ganty, Majumdar ’09], they consider timed constraints on tasks but the model is different from ours

They show that the safety checking for their model is undecidable

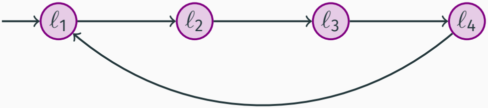
[1]Fersman et al. 2007; Norstrom et al. 1999; Fersman et al. 2002

Main contribution

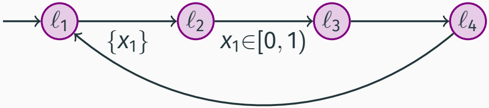


Model

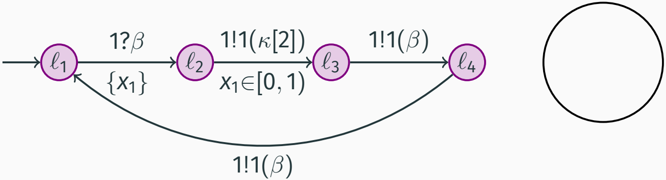
Automata



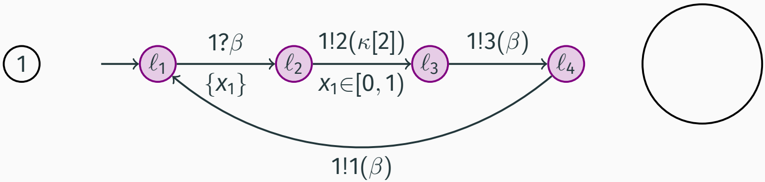
Timed Automata



Multiset Timed Automata



N - Multiset Timed Automata



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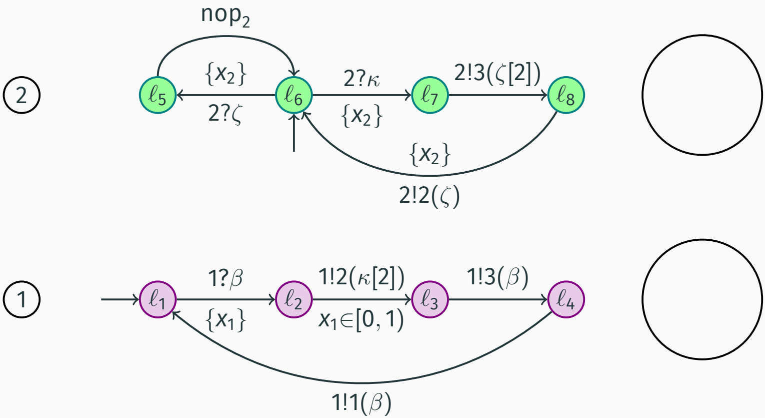

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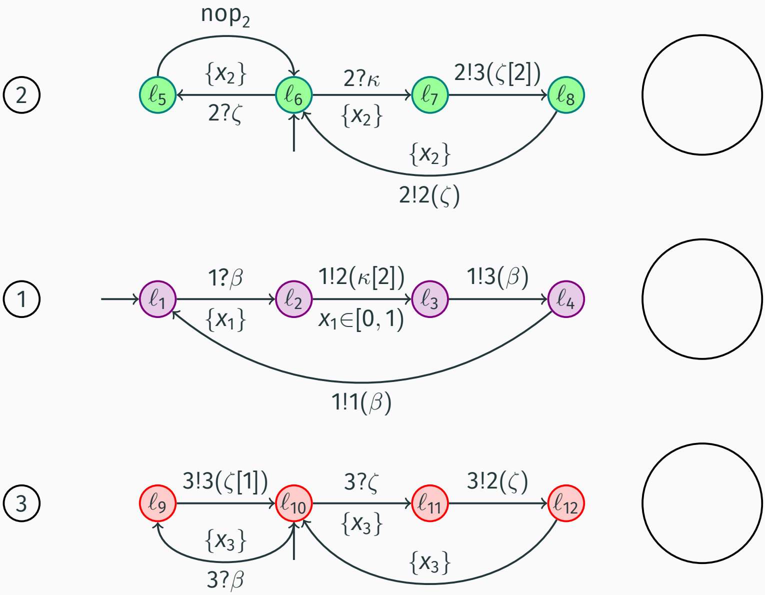


Configuration of N-MTA

Configurations of the Timed Automata
(state + clock valuation)

+

Multiset Configurations

Verification Problems

Two Problems

Control State Reachability

Two Problems

Control State Reachability

Configuration Reachability

Two Problems

Control State Reachability

Can a particular tuple of states be reached?

Configuration Reachability

Two Problems

Control State Reachability

Can a particular tuple of states be reached?

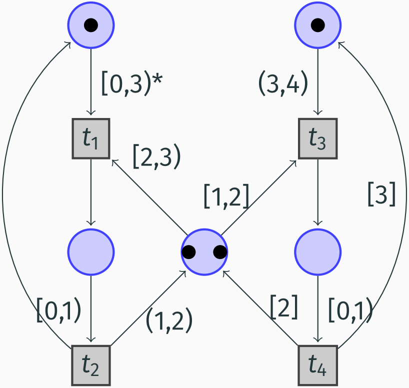
Configuration Reachability

Can a particular tuple of states with empty multisets be reached?

Control State Reachability

Claim. Control State Reachability is decidable for N-MTA
Idea. Reduction to the coverability problem for Timed Petri Nets with read-arcs (RTPN)

Timed Petri Nets with read arcs



Reduction from N-MTA to RTPN

States $\longleftrightarrow$ one place each

To simulate the state of the automata, only one place per automata is marked at a time

Reduction from N-MTA to RTPN

States	$\longleftrightarrow$	one place each
Clocks	$\longleftrightarrow$	one place each

Reduction from N-MTA to RTPN

States	$\longleftrightarrow$	one place each
Clocks	$\longleftrightarrow$	one place each
Bags	$\longleftrightarrow$	multiple places for each

A place for each multiset, for each action, for each (integer) deadline value: $\{0, 1, \ldots, d_{max}, \infty\}$

Reduction from N-MTA to RTPN

States	$\longleftrightarrow$	one place each
Clocks	$\longleftrightarrow$	one place each
Bags	$\longleftrightarrow$	multiple places for each
Transitions	$\longleftrightarrow$	Normal arcs

When picking a task, check deadline as constraint on arc

Reduction from N-MTA to RTPN

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Clocks	$\longleftrightarrow$	one place each
Bags	$\longleftrightarrow$	multiple places for each
Transitions	$\longleftrightarrow$	Normal arcs
Clock Constraints	$\longleftrightarrow$	Read arcs

Configuration Reachability

Claim. Configuration Reachability is undecidable for N-MTA
Idea. Reduction from the reachability problem for a 2-counter machine

Reduction from 2-counter machines

Reduce reachability of 2-counter machines to configuration reachability of a 1-MTA

- States of 1-MTA simulate states of the 2-counter machine

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    - Check $x = 0$

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  - Consume all tasks 2[0] and add tasks 2[1]

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  - Change state while checking $x = 0$

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  - Check $x = 1$ and reset clock
  - Consume all tasks 2[1] and add as many 2[0] back
  - Check $x = 0$, go to normal execution
- Zero tests guessed correctly if empty pending tasks at the end

Special subclass

Stateless and Time-Independent

Stateless

Stateless and Time-Independent

Stateless **Time-independent**

Stateless and Time-Independent

Stateless

Unique state per automata for picking up a task

Time-independent

Stateless and Time-Independent

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Unique state per automata for picking up a task

Time-independent
Clocks reset before picking up a task

Control State Reachability

Claim. Control State Reachability is PSPACE-complete for stateless & time-independent N-MTA

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- Reduction to a PSPACE-complete problem: coverability of 1-safe RTPNs

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- Number of *relevant* tasks in any bag at any point of time in the run is bounded

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Reduction to a PSPACE-complete problem: coverability of 1-safe RTPNs

- Number of *relevant* tasks in any bag at any point of time in the run is bounded
- Once bound is established, construct 1-safe RTPN

Bounding the number of relevant tasks

Run. Sequence of time-elapse and discrete transitions

Bounding the number of relevant tasks

Given a run σ, define σ_i as the sequence of transitions corresponding to a particular automata i

Block in σ_i. Transitions following picking up a task before picking up the next task

Bounding the number of relevant tasks

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Label all transitions with its block label

Bounding the number of relevant tasks

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Relevance of tasks

- Since we only care about the final control state that is reached, if a block does not affect the final state of any automata, we can ignore it
- If a control state can be reached starting from a configuration, it can also be reached starting from a *larger* configuration
- Starting backwards, one can construct a dependency graph i.e. which blocks affect the final state

Bounding the number of relevant tasks

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- This is because the relevance of blocks is based on *which task added which task* which eventually resulted in the final control state
- There cannot be more than one task in the bag whose blocks resulted in the final control state of the same automata

Reduction to 1-safe RTPN

Similar to the previous construction of RTPN but ...

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- Since the number of relevant tasks is bounded, we have multiple 1-safe places to store them
- While picking a task, can pick from any of the N copies
- While adding a task, non-deterministically add to any one of the multiple places
- Choose to not add a task non-deterministically (guess it to be not *relevant*)

Configuration Reachability

Claim. Configuration Reachability is decidable for stateless & time-independent N-MTA
Idea.

WQO over the configurations of the N-MTA

Karp-Miller style algorithm for reachability

Regions

- Potentially infinite configurations (state + clock valuation + multiset config)

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$$\begin{aligned}\text{Region} = &\ (\text{clocks} + \text{tasks with fractional part} = 0)\\ &+ (\text{clocks} + \text{tasks with smallest fractional part})\\ &+ (\text{clocks} + \text{tasks with second smallest fractional part})\\ &+ \dots\\ &+ (\text{clocks} + \text{tasks with ages larger than the } \textit{max} \text{ value})\end{aligned}$$

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- Regions form a WQO

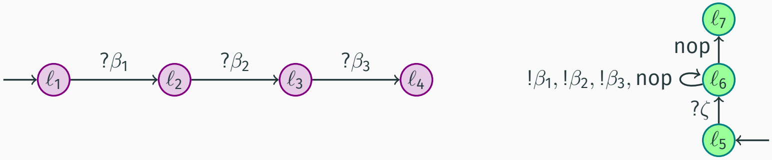
Algorithm for decidability

- Start with initial region and add it to a set
- Pick an unmarked region from the set, add its successors to the set and mark the current region
- In the set, at any point, if there is a region larger than another region in the set, remove it

Termination guaranteed from the WQO property of regions

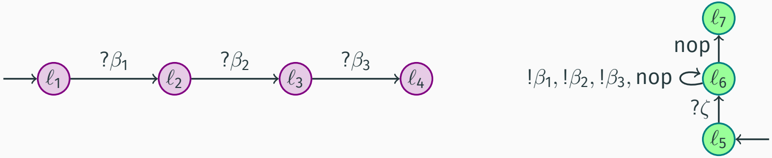
Works because if an empty multiset can be reached from a configuration, it can also be reached from a smaller configuration

Statelessness



2-MTA which is not stateless

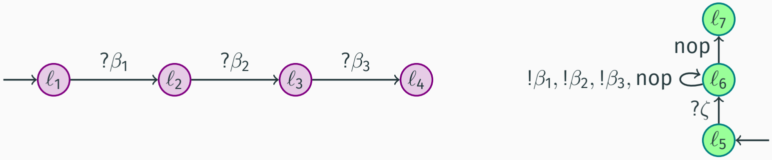
Statelessness



$\boxed{\text{2-MTA which is not stateless}}$

$$\mathfrak{c}_1 = (\ell_1, \ell_6, \{\beta_1, \beta_3\}, \emptyset) \preceq (\ell_1, \ell_6, \{\beta_1, \beta_2, \beta_3\}, \emptyset) = \mathfrak{c}_2$$

Statelessness



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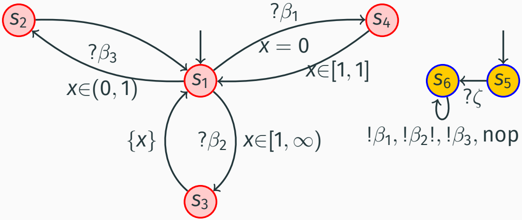
From $\mathfrak{c}_2$, one can reach $(\ell_4, \ell_6, \emptyset, \emptyset)$, but not from $\mathfrak{c}_1$

Time-independence



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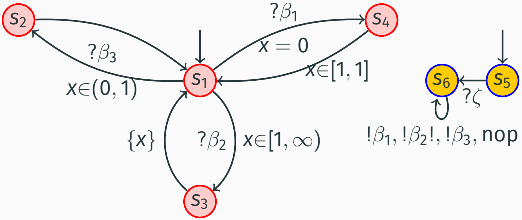
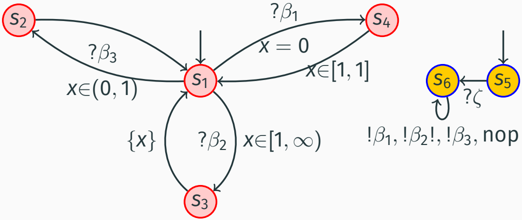


2-MTA which is not time independent

$$\mathfrak{c}_1 {=} (((s_1, 0), s_6), \{(\beta_1, 0, \infty), (\beta_3, 0, \infty)\}, \emptyset)$$

Time-independence



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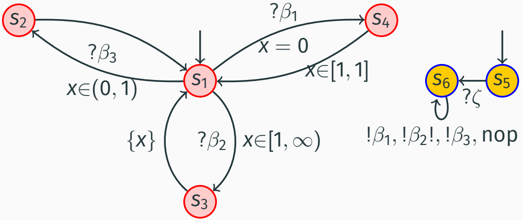

2-MTA which is not time independent

$\mathfrak{c}_1 = (((s_1, 0), s_6), \{(\beta_1, 0, \infty), (\beta_3, 0, \infty)\}, \emptyset)$

$\mathfrak{c}_2 = (((s_1, 0), s_6), \{(\beta_1, 0, \infty), (\beta_2, 0, \infty), (\beta_3, 0, \infty)\}, \emptyset)$ $\mathfrak{c}_1 \preceq \mathfrak{c}_2$

$(((s_1, 0), s_6), \emptyset, \emptyset)$ is reachable from $\mathfrak{c}_2$ but not from $\mathfrak{c}_1$.

Conclusion

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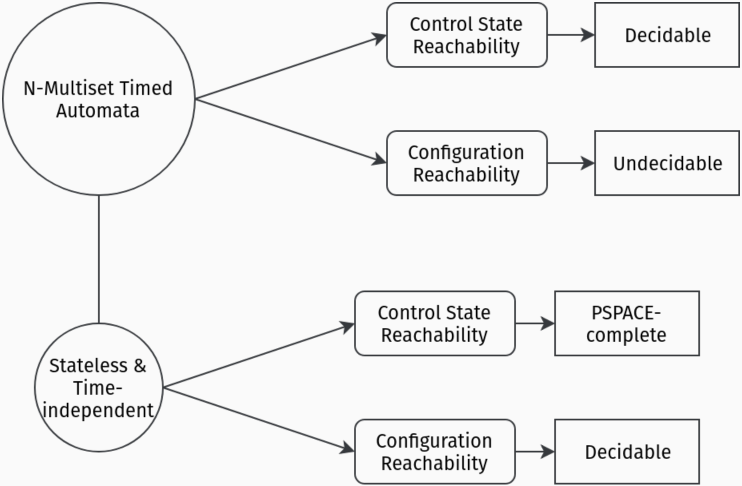

N-Multiset Timed Automata
Control State Reachability
Decidable
Configuration Reachability
Undecidable
Stateless & Time-independent
Control State Reachability
PSPACE-complete
Configuration Reachability
Decidable

Future Work

Priority for tasks

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Recursive programs: Timed pushdown automata

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Schedulability?

Questions?